UE5 Fundamentals of Algorithms

Lecture 2: Recursion

Ecole Centrale de Lyon, Bachelor of Science in Data Science for Responsible Business

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Outline

- What is recursion?
- Recursive/iterative approach
- Recursive data structures
- Tail vs non-tail recursion

What is recursion?

> *A function is considered **recursive** when it invokes itself. This self-invocation typically involves handling a straightforward case, known as the **base case**, and implementing instructions that lead towards reaching this base case.*

Why recursion?

- Main reason: some problems are more easily implementable recursively.
- Other reeasons: Iterative code is difficult to parallelize
- Code can be more elegant and concise than the iterative version, simpler to implement.
- Similarity to proof by induction.

Example 0: countdown

In [37]:
```python
def countdown(n):
    if n <= 0: # base case
        print("Blastoff!")
    else:
        print(n)
        countdown(n - 1) # recursive call

countdown(5)
```

```
5
4
3
2
1
Blastoff!
```

Here's how the countdown(5) call would work:

```
countdown(5) prints 5 and calls countdown(4)
countdown(4) prints 4 and calls countdown(3)
countdown(3) prints 3 and calls countdown(2)
countdown(2) prints 2 and calls countdown(1)
countdown(1) prints 1 and calls countdown(0)
countdown(0) prints "Blastoff!" and returns without making another recursive call.
```

Example 1: factorial (iterative)

Reminder: here is the factorial function in the iterative version.

```
In [1]:
def factorial_iter(n):
    r = 1
    for i in range(1, n + 1):
        r *= i
    return r
```

```
In [14]:
assert factorial_iter(0) == 1
assert factorial_iter(1) == 1
assert factorial_iter(5) == 120
assert factorial_iter(10) == 3628800
```

- The function `fact_iter` is called once
- Uses iterative operator (loop, while, etc.)

Example 1: factorial (recursive)

Write a recursive version of the factorial function.

Example 1: factorial (recursive)

Write a recursive version of the factorial function.

In [2]:
```
def factorial_rec(n):
    if n == 0:
        return 1
    else:
        return n * factorial_rec(n - 1)
```

In [3]:
```
assert factorial_iter(0) == factorial_rec(0)
assert factorial_iter(1) == factorial_rec(1)
assert factorial_iter(5) == factorial_rec(5)
assert factorial_iter(10) == factorial_rec(10)
```

- The function `factorial_rec` is called multiple times
- Identical results to an *iterative* version of the function

Example: Sum of digits

Write the recursive version of the sum of digits.

In [12]:
```
def iterative_sum(arr):
    total = 0
    for num in arr:
        total += num
    return total
```

In [13]:
```
iterative_sum(range(4))
```

Out[13]:
```
6
```

Example: Sum of digits

Write the recursive version of the sum of digits.

```
In [12]: def iterative_sum(arr):
             total = 0
             for num in arr:
                 total += num
             return total
```

```
In [13]: iterative_sum(range(4))
```

```
Out[13]: 6
```

```
In [14]: def recursive_sum(arr):
             if not arr:
                 return 0
             else:
                 return arr[0] + recursive_sum(arr[1:])
```

```
In [15]: assert iterative_sum(range(10)) == recursive_sum(range(10))
```

Example 2: Fibonnacci (iterative)

```
In [16]: def fibo_iter(n):
             arr = [0, 1]
             for i in range(2, n+1):
                 arr.append(arr[i-1] + arr[i-2])
             return arr[n]
```

```
In [17]: fibo_iter(4)
```

```
Out[17]: 3
```

Recursive version?

Example 2: Fibonnacci (iterative)

```
In [16]: def fibo_iter(n):
             arr = [0, 1]
             for i in range(2, n+1):
                 arr.append(arr[i-1] + arr[i-2])
             return arr[n]
```

```
In [17]: fibo_iter(4)
```

```
Out[17]: 3
```

Recursive version?

```
In [18]: def fibo_rec(n):
             if n <= 1:
                 return n
             else:
                 return fibo_rec(n-1) + fibo_rec(n-2)
```

```
In [19]: assert fibo_rec(10) == fibo_iter(10)
```

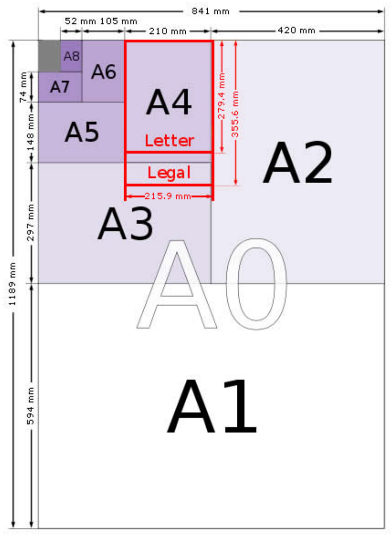
Recursion in real life (plants)

https://en.wikipedia.org/wiki/Barnsley_fern
https://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension

Recursion in real life (paper folding)

https://en.wikipedia.org/wiki/Paper_size

(uses the ISO 216 standard's scaling factor of approximately 1.1892071 (2^(1/4)) to calculate the dimensions of various ISO paper sizes based on their relation to A0)



Recursion in real life (paper folding) in Pseudo Code

1. Constant Ratio:
    - ratio = $\sqrt{2}$
2. Creating A_i from A_i-1:
    - function createPaper(A_i-1):
        - fold A_i-1 along its length to create A_i
3. Recurrence Relation:
    - function calculateDimensions(A_i-1):
        - length_A_i = width_A_i-1
        - width_A_i = length_A_i-1 / 2

Recursion in real life (paper folding) in Python

In [20]:
```
def generate_iso_paper_sizes(n):
    iso_sizes = {"A0": (841, 1189)}
    current_size = "A0"

    for i in range(1, n + 1):
        width, height = iso_sizes[current_size]
        next_size = f"A{i}"
        iso_sizes[next_size] = (height / 2, width)
        current_size = next_size

    return iso_sizes
```

```
A0: Width = 841 mm, Height = 1189 mm
A1: Width = 594.5 mm, Height = 841 mm
A2: Width = 420.5 mm, Height = 594.5 mm
A3: Width = 297.25 mm, Height = 420.5 mm
A4: Width = 210.25 mm, Height = 297.25 mm
A5: Width = 148.625 mm, Height = 210.25 mm
```

In [21]:
```
iso_paper_sizes = generate_iso_paper_sizes(5)
for size, dimensions in iso_paper_sizes.items():
    print(f"{size}: Width = {dimensions[0]} mm, Height = {dimensions[1]
```

```
A0: Width = 841 mm, Height = 1189 mm
A1: Width = 594.5 mm, Height = 841 mm
A2: Width = 420.5 mm, Height = 594.5 mm
```

```
A3: Width = 297.25 mm, Height = 420.5 mm
A4: Width = 210.25 mm, Height = 297.25 mm
A5: Width = 148.625 mm, Height = 210.25 mm
```

Recursive data structures

Some **data types** are inherently recursive, meaning that a subset of this data type has the same data type:

- Lists
- Strings (arrays)
- Binary trees
- Linked lists
- Custom (e.g., using objects)

Tail vs non-tail recursion

Recursion can be categorized into two types: **tail recursion** and **non-tail recursion**.

- **tail recursion**, the recursive call is the last operation before returning a result. This makes it efficient in terms of memory usage and stack overflow risk, as there are no pending calculations.

- **non-tail recursion** involves additional operations after the recursive call, potentially leading to a stack of calls and increased memory consumption.

The choice between these types depends on the problem and programming language. Tail recursion often leads to more efficient and readable code, but sometimes non-tail recursion is necessary, impacting variable lifespan and the result.

⚠ Impacts results and variables span of life

Tail vs non-tail recursion (cont.)

In [42]:
```
def print_desc(n):
    if n > 0:
        print(" " * n, "  n= " , n ," brefore recursive call")
        print_desc (n - 1)
```

In [43]:
```
print_desc(3)
```

```
      n=   3  brefore recursive call
     n=   2  brefore recursive call
    n=   1  brefore recursive call
```

Tail vs non-tail recursion (cont.)

```
In [42]: def print_desc(n):
             if n > 0:
                 print(" " * n, "  n= " , n ," brefore recursive call")
                 print_desc (n - 1)
```

```
In [43]: print_desc(3)
```

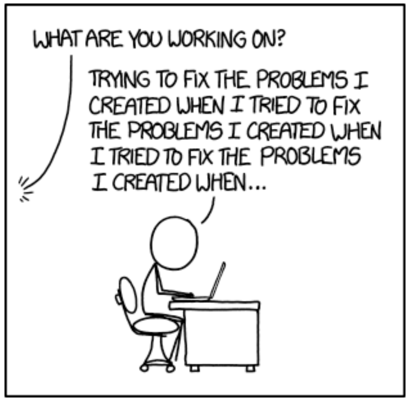
```
       n=  3  brefore recursive call
      n=  2  brefore recursive call
     n=  1  brefore recursive call
```

```
In [44]: def print_asc(n) :
             if n > 0:
                 print_asc(n - 1)
                 print(" " * n, "  n= " , n ," after recrusive call")
```

```
In [45]: print_asc(3)
```

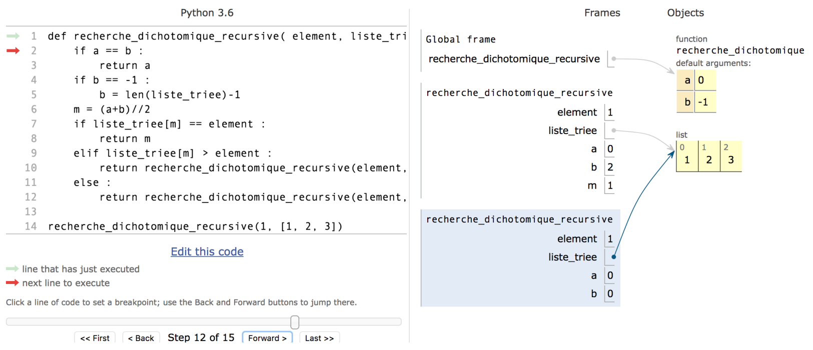
```
     n=  1  after recrusive call
      n=  2  after recrusive call
       n=  3  after recrusive call
```

Disadvantages of Recursion


WHAT ARE YOU WORKING ON?
TRYING TO FIX THE PROBLEMS I CREATED WHEN I TRIED TO FIX THE PROBLEMS I CREATED WHEN I TRIED TO FIX THE PROBLEMS I CREATED WHEN...

- While recursion can lead to concise and elegant solutions, it's crucial to handle base cases properly to avoid infinite loops or excessive function calls.
- Recursive functions require careful consideration of termination conditions to prevent stack overflow errors or excessive memory usage.
- Can become memory-intensive if written poorly.
- One must wait for the base case to start getting results (no intermediate result), results cannot be obtained progressively (if non-tail recursion).

Recursion calls


Python 3.6

```
def recherche_dichotomique_recursive( element, liste_tri
    if a == b :
        return a
    if b == -1 :
        b = len(liste_triee)-1
    m = (a+b)//2
    if liste_triee[m] == element :
        return m
    elif liste_triee[m] > element :
        return recherche_dichotomique_recursive(element,
    else :
        return recherche_dichotomique_recursive(element,

recherche_dichotomique_recursive(1, [1, 2, 3])
```

```
Global frame
recherche_dichotomique_recursive

recherche_dichotomique_recursive
  element  1
  liste_triee
  a  0
  b  2
  m  1

recherche_dichotomique_recursive
  element  1
  liste_triee
  a  0
  b  0
```

```
function
recherche_dichotomique
default arguments:
  a  0
  b  -1

list
  0  1  2
  1  2  3
```

Frames
Objects
Edit this code
line that has just executed
next line to execute
Click a line of code to set a breakpoint; use the Back and Forward buttons to jump there.
<< First
< Back
Step 12 of 15
Forward >
Last >>

Visualize recursive calls pythontutor

In case of too many function calls, a **maximum recursion depth exceeded error** is triggered. You can anticipate this error by not exceeding the limit `sys.getrecursionlimit()` or by changing it using `sys.setrecursionlimit`.

In [30]:
```python
import sys
sys.getrecursionlimit()
```

Out[30]:
```
3000
```

Transformation Schemes

There are some recursive to iterative transformation schemes:

1. Write a recursive algorithm.
2. Test, validate, and prove it (in critical applications).
3. Apply transformation techniques to obtain an iterative version.

Note that these techniques are not 100% automated.

```
f_rec(X) =
  if p(X) then a(X)
  else
    b(X)
    f_rec(new(X))
  end if;

f_iter(X) =
  if p(X) = false then
    do
      b(X);
      X := new(X);
    until p(X) = true;
  end if;
  a(X);
```