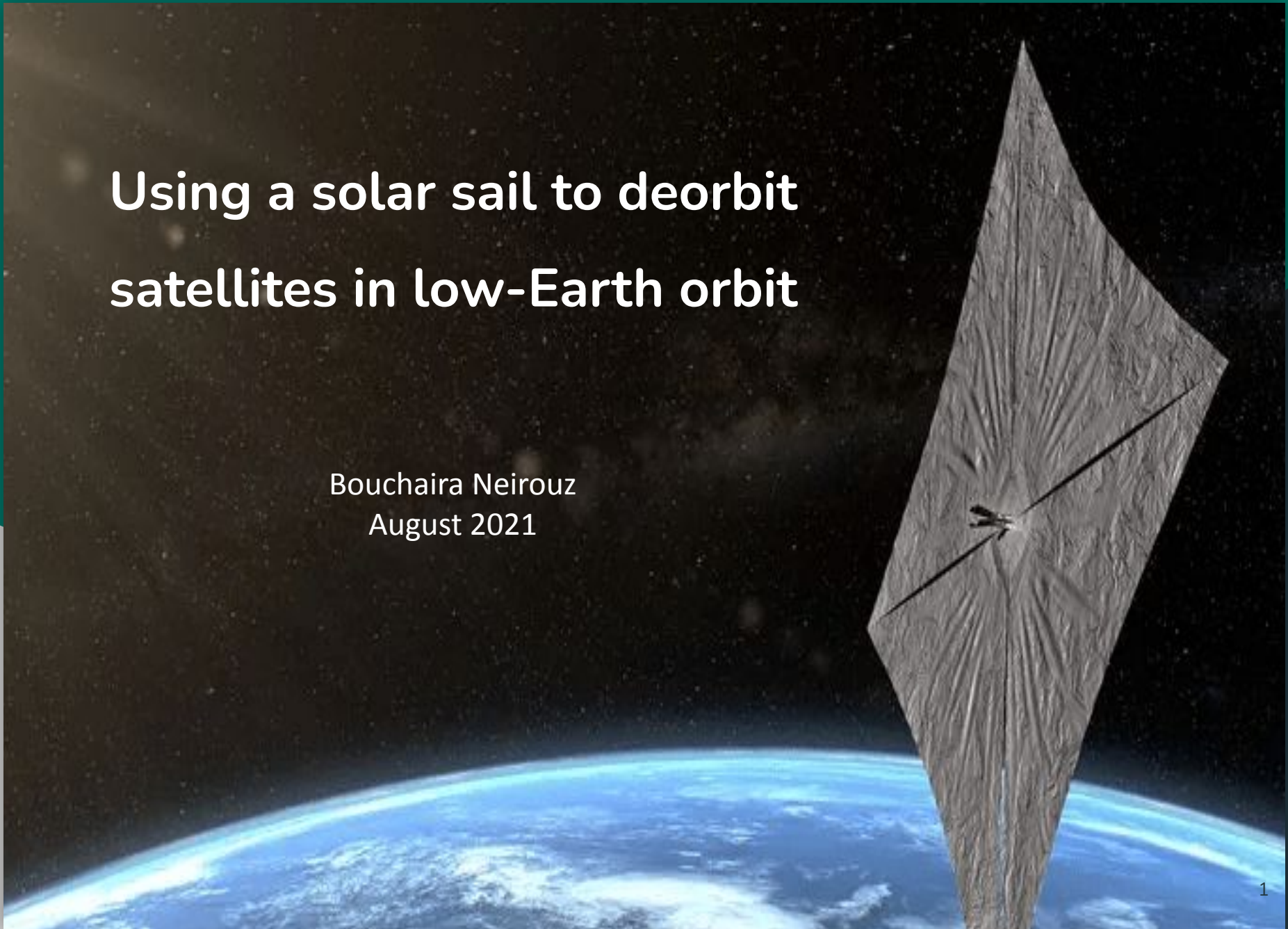


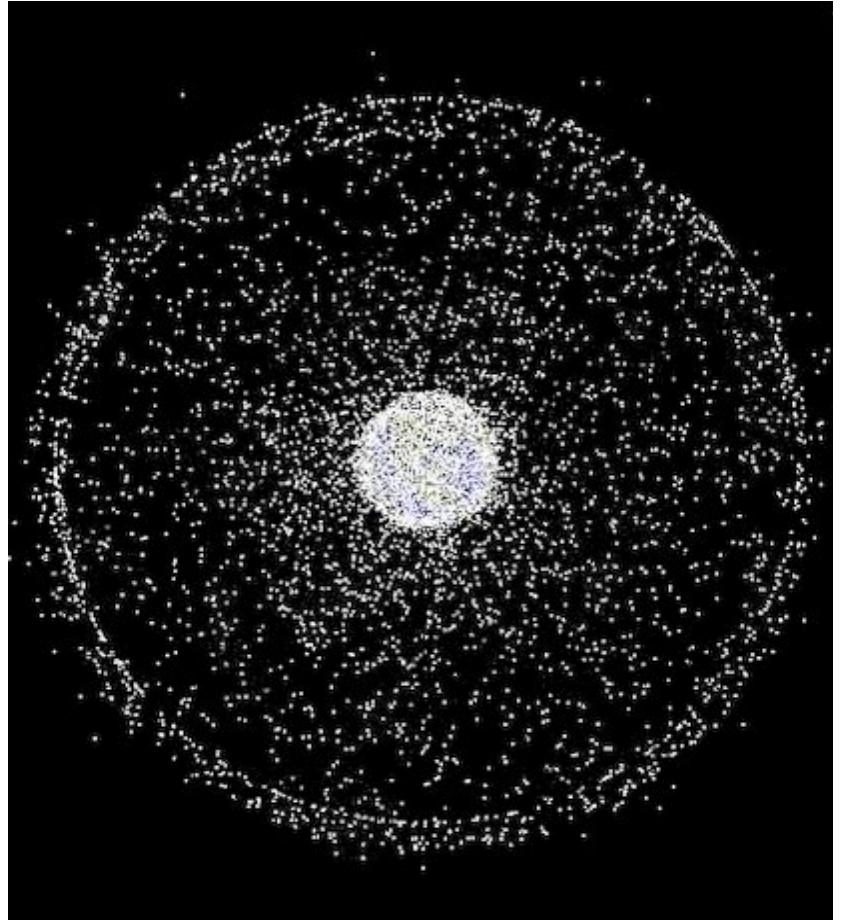
# Using a solar sail to deorbit satellites in low-Earth orbit

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August 2021



# Introduction

- Space pollution
- Threat to scientific research
- Societal issues
- De-orbiting methods
- Solar sails



# Problematic

How can solar sails be used to reduce the time satellites  
spend in the atmosphere?

# Table of contents

- Satellite trajectory
  - Model
  - Equations
- Solar sails
  - Principle
  - Modeling the radiation force
- Application
  - Application to a concrete case
  - Modeling and comparison
  - Limits

# Satellite trajectory

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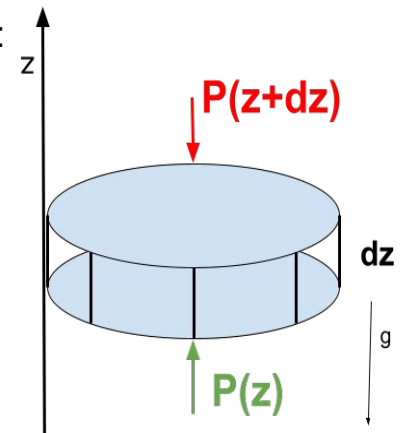
# Modèle

- Geocentric reference frame assumed to be Galilean
- polar coordinates (plane trajectory)
- Assumptions:
  - Undeformable solid
  - Constant mass
  - Thermosphere 80 km-700 km
  - Average temperature: 0°C

## Exponential law of air density:

Hydrostatic equilibrium :

$$\frac{dp}{dz} + \rho g = 0$$



$$\rho = \frac{P_0 M}{RT} e^{-\frac{z}{H_0}} \quad \text{with} \quad H_0 = \frac{R T_0}{Mg}$$

# Equations

## Force balance:

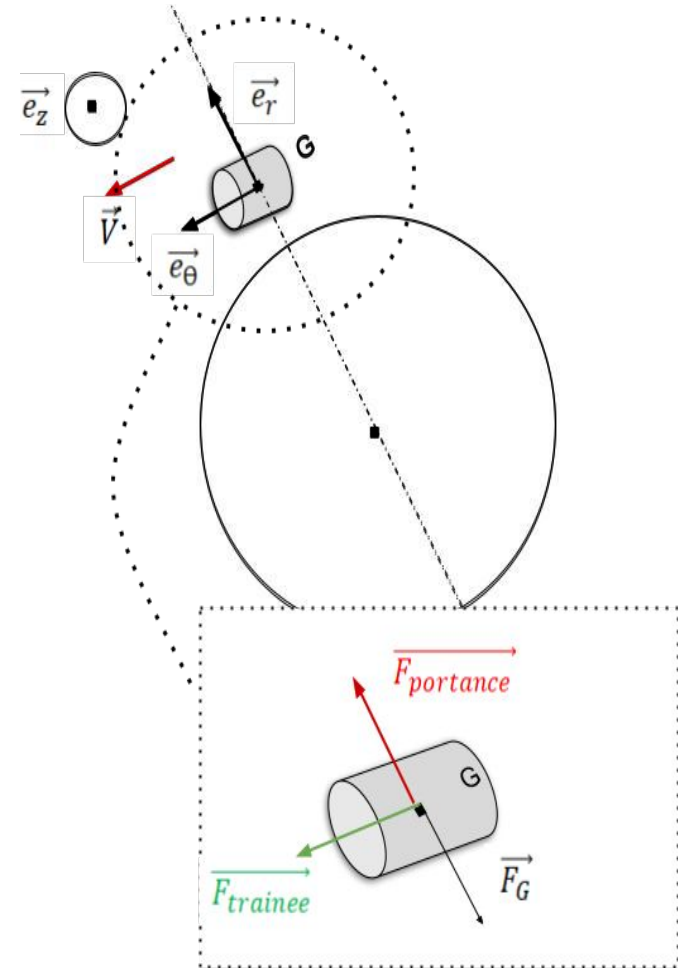
Lift force: 
$$\overrightarrow{F_{portance}} = \frac{1}{2} \rho V S C_p \overrightarrow{V^\perp}$$

Drag force: 
$$\overrightarrow{F_{trainee}} = -\frac{1}{2} \rho V S C_t \overrightarrow{V}$$

Gravitational attraction: 
$$\overrightarrow{F_G} = -\frac{G M_t m}{r^2} \overrightarrow{e_r}$$

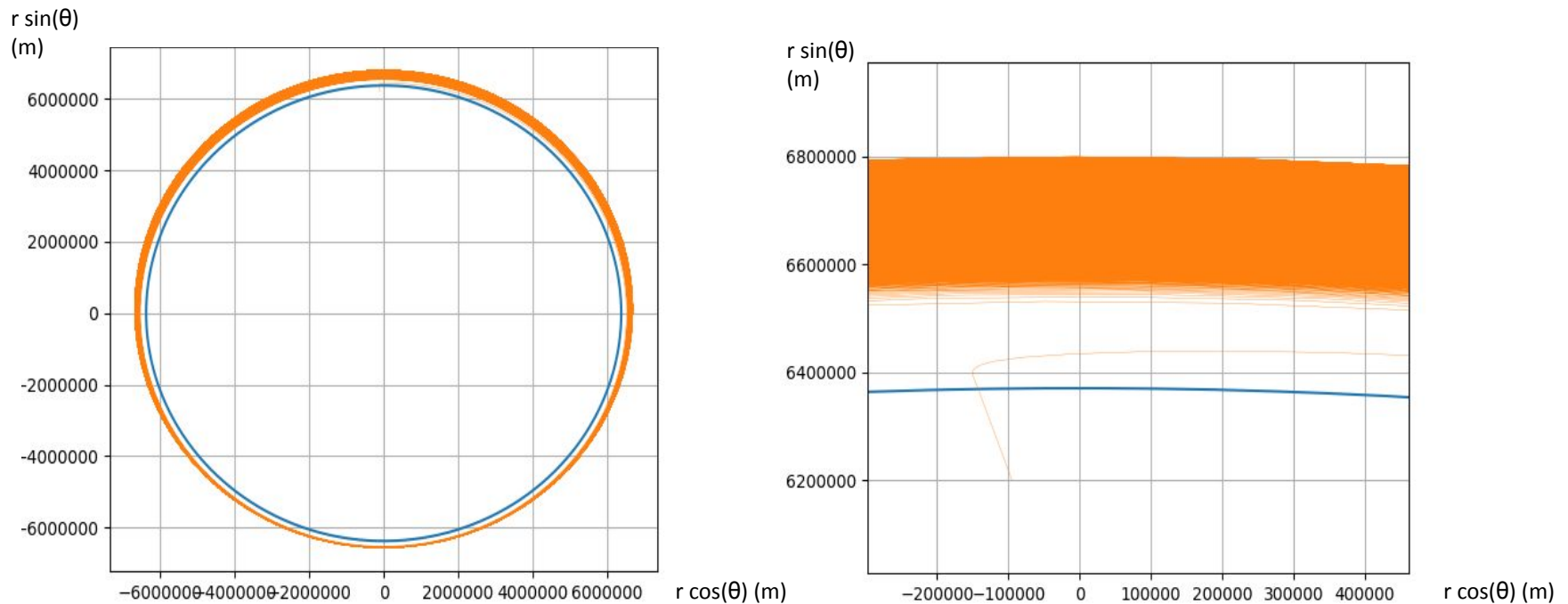
## Fundamental relationship of dynamics:

$$m \frac{d\overrightarrow{V}}{dt} = \frac{1}{2} \rho V S C_p \overrightarrow{V^\perp} - \frac{1}{2} \rho V S C_t \overrightarrow{V} - \frac{G M_t m}{r^2} \overrightarrow{e_r}$$



$$\begin{cases} m(\ddot{r} - r\dot{\theta}^2) = \frac{1}{2} \rho V S (C_p r \dot{\theta} - C_t \dot{r}) - \frac{G M_t m}{r^2} \\ m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = -\frac{1}{2} \rho V S (C_t r \dot{\theta} - C_p \dot{r}) \end{cases}$$

with  $V = \sqrt{\dot{r}^2 + r^2 \dot{\theta}^2}$



**Figure 1: Satellite trajectory subjected to Python friction forces**



# Solar sails

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# Principle

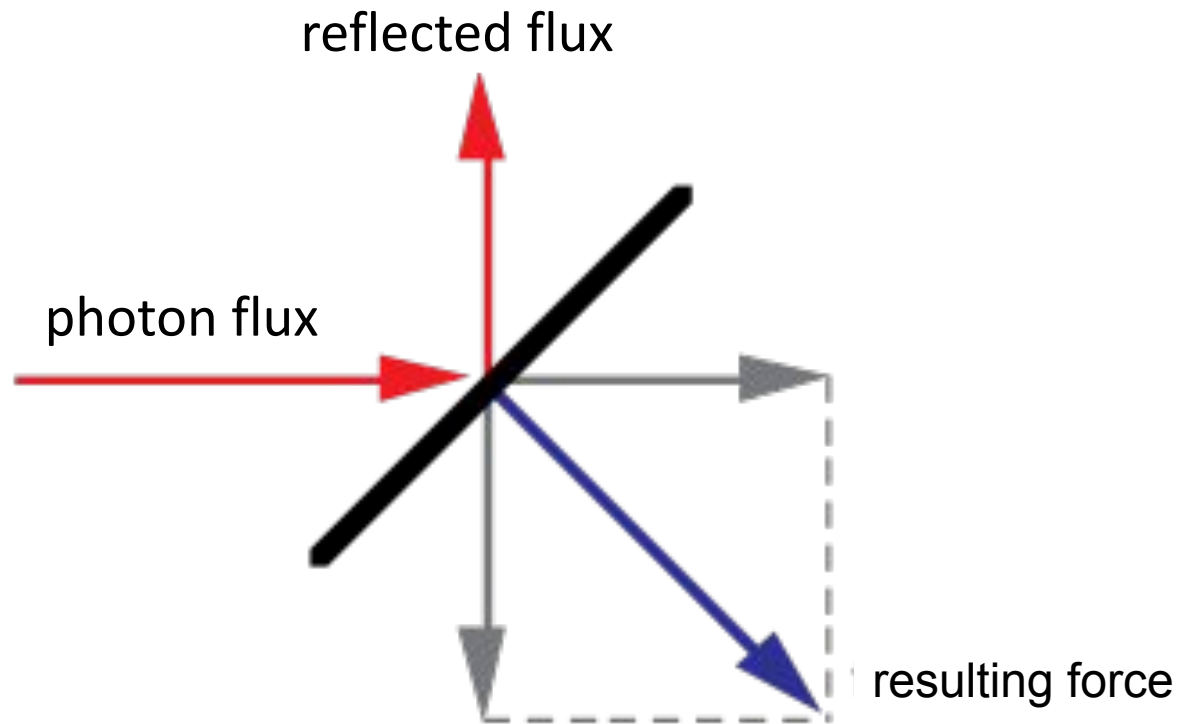
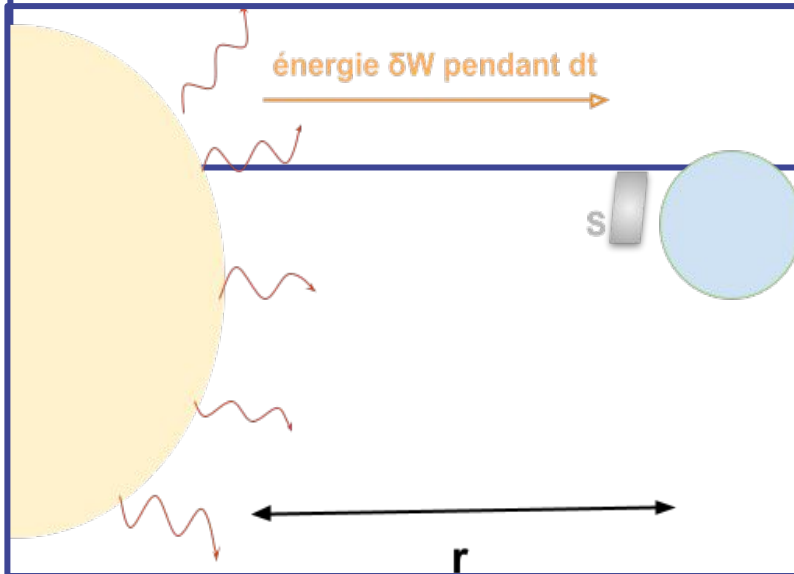


Figure 2: Thrust force of solar radiation

# Modélisation de la force de radiation

Where does the power emitted by the sun come from?

$$P_0 = 4\pi R_s^2 \sigma T^4 \quad \Rightarrow \quad P_0 = 3,85 \cdot 10^{26} \text{ W}$$



F radial repulsion, due to radiation pressure

$$\vec{F} = \frac{1}{c} \frac{\partial W}{\partial t} \vec{e}_r$$

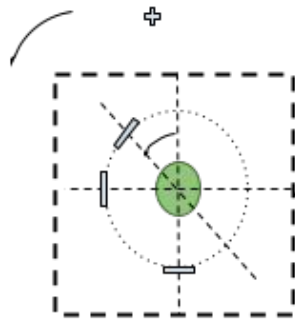
The power emitted by the sun is distributed over a sphere of radius  $r$

$$\delta W = P_0 \frac{S}{4\pi r^2} dt \quad \text{donc} \quad F = P_0 \frac{S}{4\pi c r^2}$$

Numerical application:

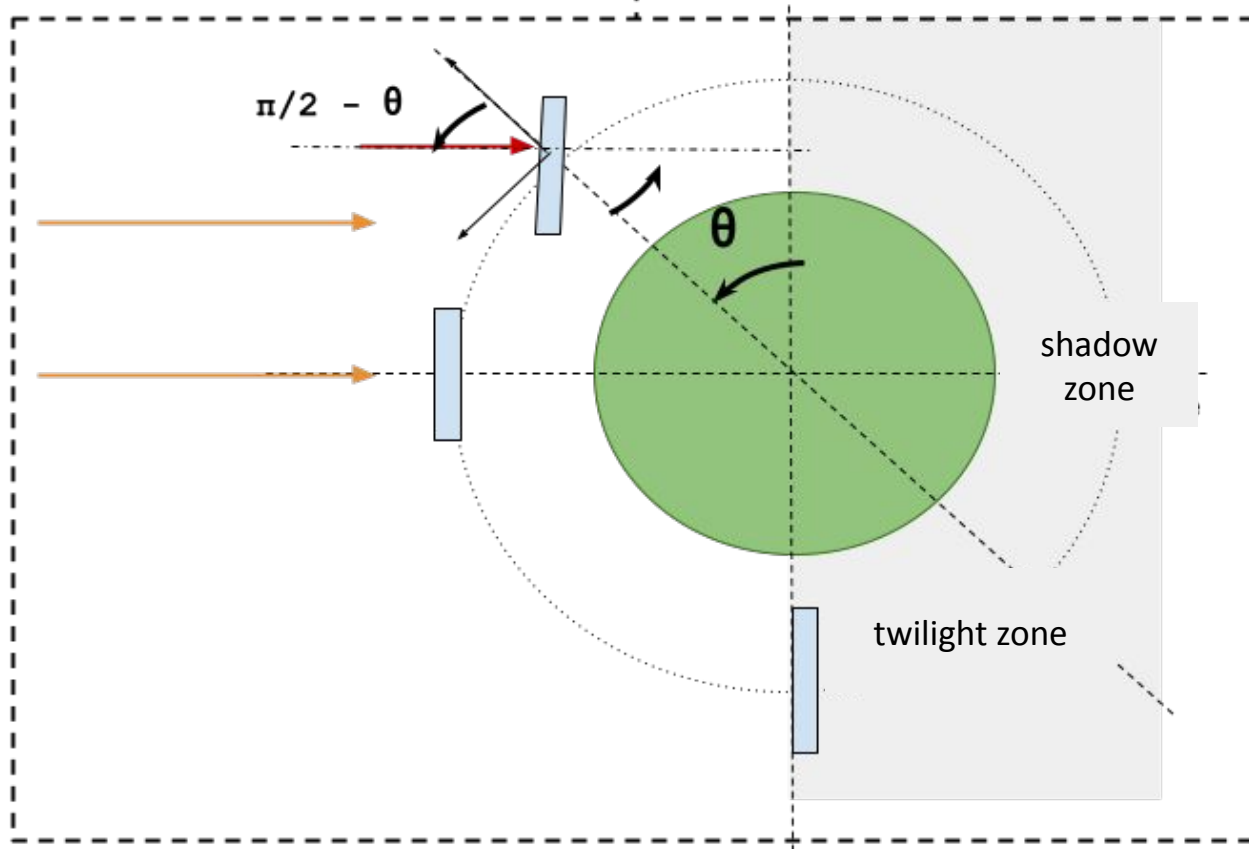
for a sail model similar to Nasa's Sunjammer  $S=1200\text{m}^2$ , we find:

$$F=10^{-2}\text{N}$$



$$\vec{F} = -F \cos\left(\frac{\pi}{2} - \theta\right) \vec{e}_r - F \sin\left(\frac{\pi}{2} - \theta\right) \vec{e}_\theta$$

so:  $\vec{F} = -F \sin(\theta) \vec{e}_r - F \cos(\theta) \vec{e}_\theta$



# Application

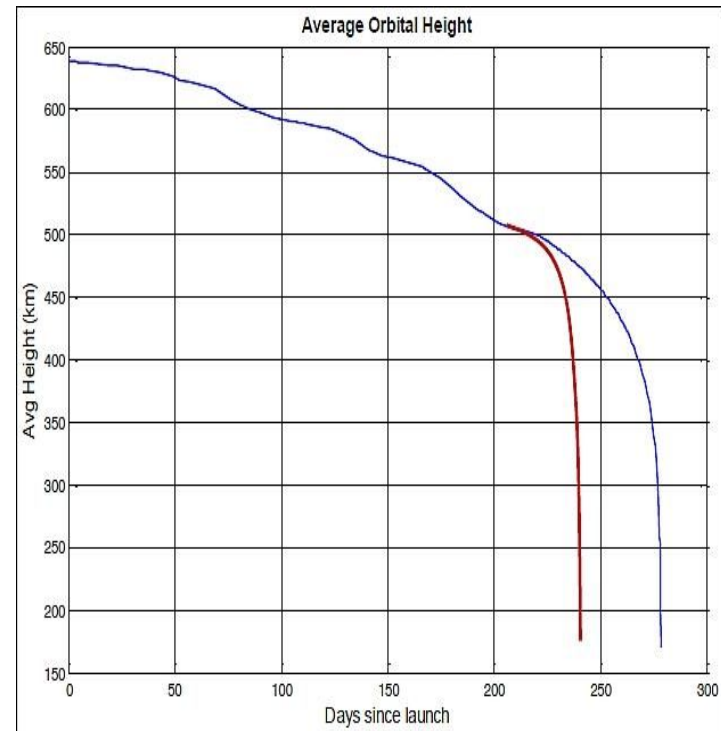
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# Application to a concrete case

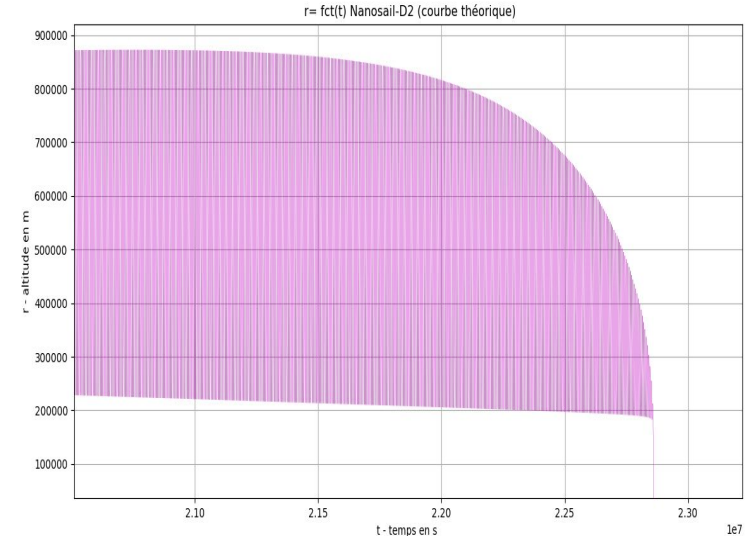
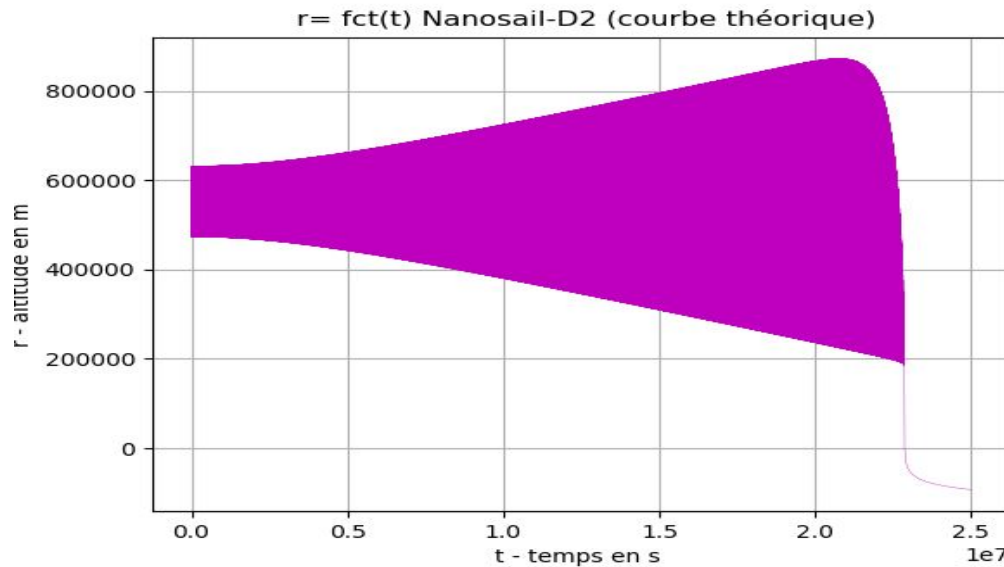
Material: NanoSail-D2



**Figure 3: NanoSail-D2 ground deployment test (Source: NASA/MSFC)**



**Figure 4: Orbital profile of the NanoSail-D2 mission (Source: NASA)**



**Figure 5: Altitude versus time for the Nanosail-D2 Python satellite**

$$T = 2,86 * 10^7 s \quad \longrightarrow \quad T = 264 \text{ jours}$$

Difference of 24 days, i.e. an error of 10% for this example

Acceptable modeling → approved

# Modélisation et comparaison

## Satellite

Mass:  $m=2000$  kg

$V_0=7635$  m/s<sup>2</sup>

Altitude= 429 km

Without  
solar sail

$t=4$  years and 5  
months (1620 days)

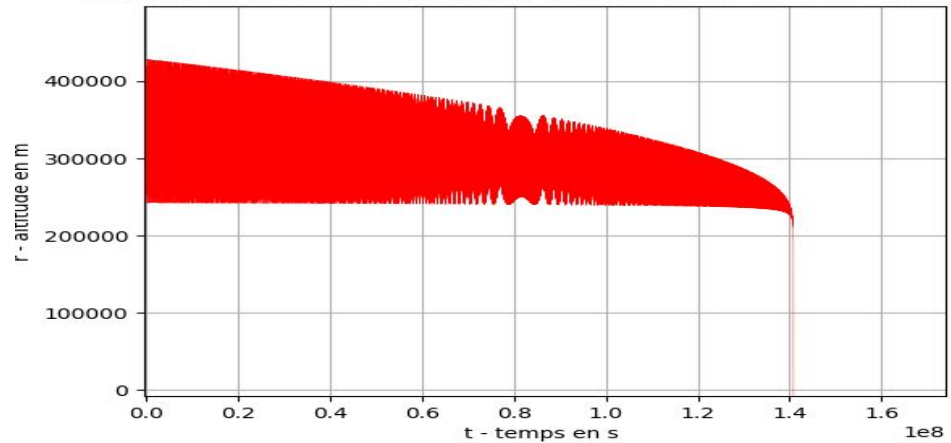


Figure 6: Altitude versus time without Python solar sail

With solar  
sail  $S=1200$   
 $m^2$

$t=284$  days

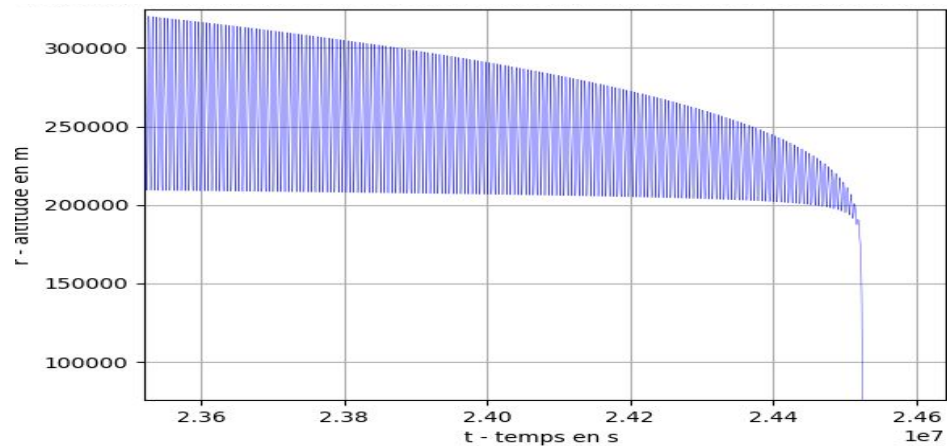


Figure 7: Altitude versus time for a sail with a surface area of 1200m<sup>2</sup> Python



t (period in days)

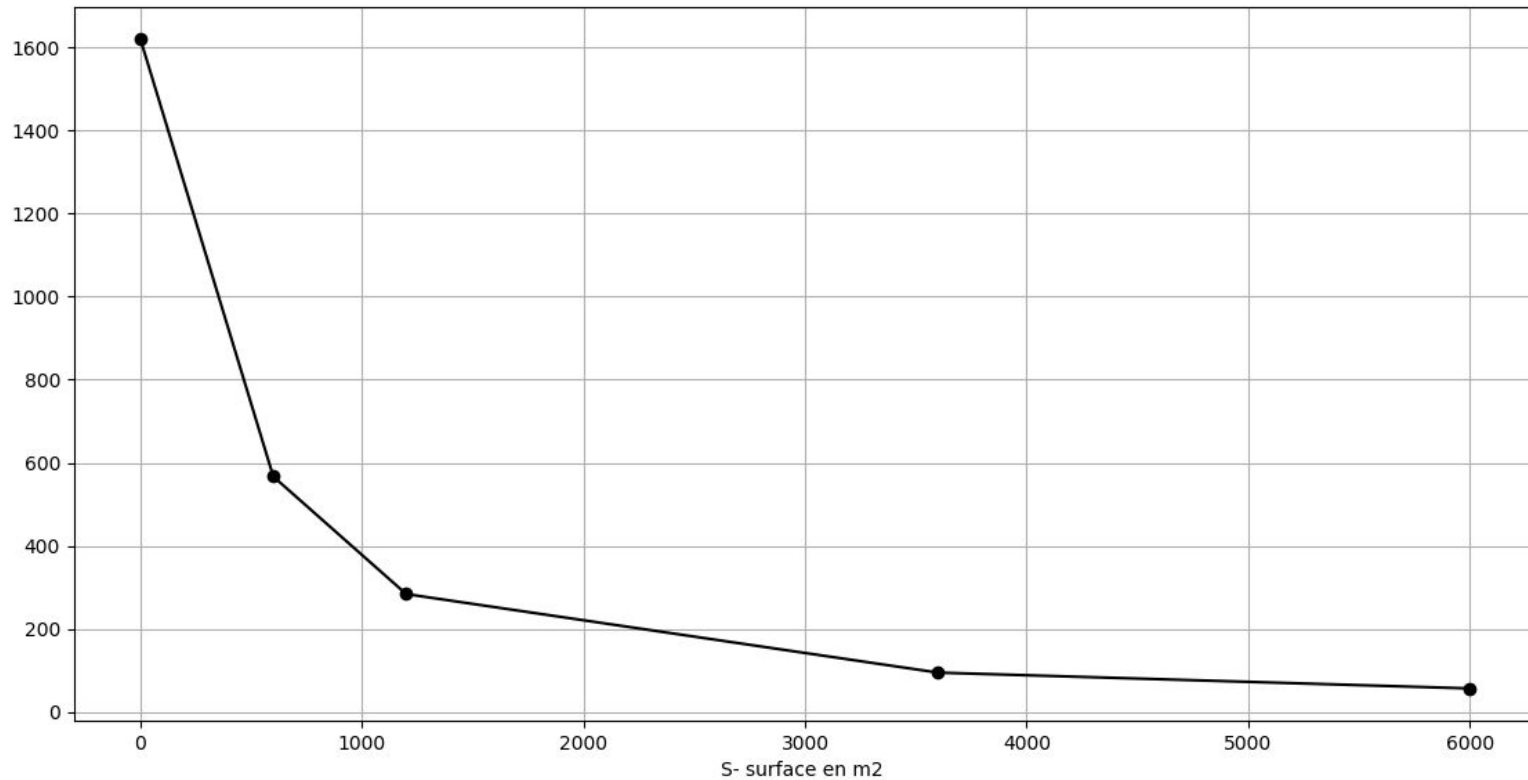
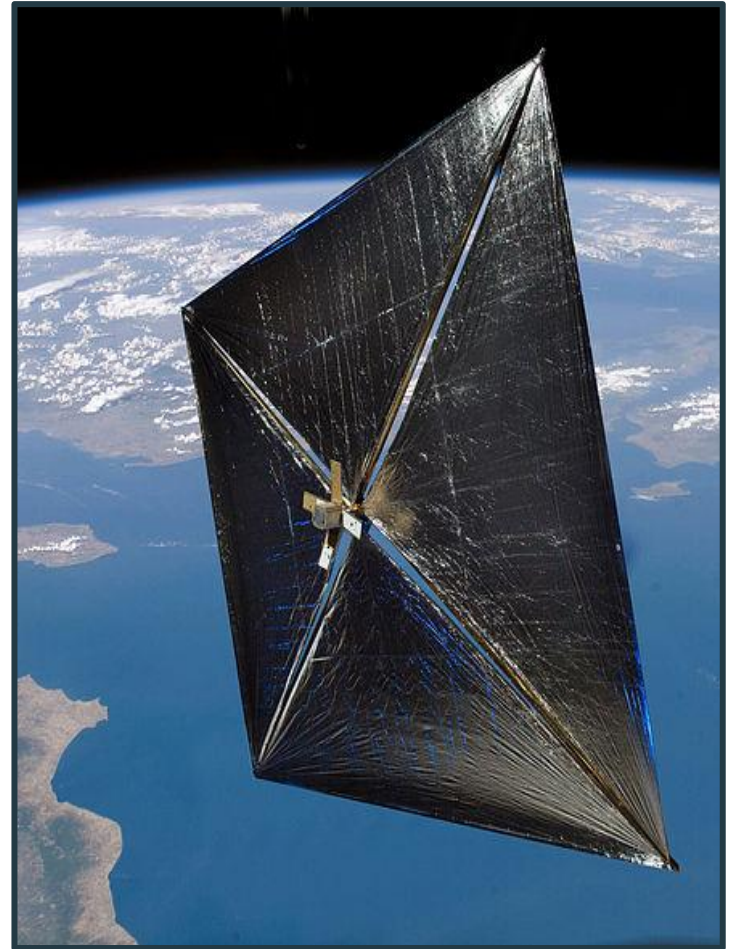


Figure 8: Orbit duration as a function of Python sail area

**Regressi:**  $T = T_0 * \exp(\frac{-S}{A})$  avec  $T_0 = 1610 \pm 230$  jours  
et  $A = 619 \pm 218 \text{ m}^2$

# Limits

- Size
- Inexhaustible energy BUT  
impossible to concentrate the Sun's  
rays on a sail to obtain sufficient  
propulsion
- Impossible to unfold the sail without  
tearing it



**Merci pour votre attention**

## Altitude versus time for a force $F=0.01N$

Appendix slides

```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
from scipy import *
from scipy.integrate import odeint
```

### #introduction des constantes

```
G = 6.7*(10**-11)
M = 6*(10**24)
m=2000
SSAT=1200 # modèle sunjammer
Cp=0.349
Ct= 1.34
Mair= 28.976* 10**(-3)
g=9.81
R=8.31
T=273.15
hs =(R*T)/(Mair*g)
```

```
def fonction(X,t):
```

```
    r = X[0]
    theta = X[1]
    v = X[2]
    omega = X[3]
    dr = v # r point
    dtheta = omega #theta point
```

# la force de radiation ne s'exerce pas sur la voile dans la zone d'ombre

```
    if theta % 2*pi > pi:
```

```
        F=0
```

```
    else:
```

```
        F=0.01
```

# projection en considérant les ondes em parallèles entre elles/ voile perpendiculaire à la direction terre-satellite

```
    dv = -(G*M)/(r**2) + r*(dtheta**2)+(
exp((6371000-r)/hs) *SSAT* sqrt(dr**2
+(r*dtheta)**2)*(Cp* r* dtheta - Ct
*dr))/(2*m)-F*sin(theta)/m # r deux points
```

```
    dw = - 2*dr*dtheta/r - ( exp((6371000-r)/hs)
*SSAT *sqrt(dr**2 +(r*dtheta)**2)*(Ct* r* dtheta -
Cp *dr) )/(2*m*r)-(F*cos(theta))/(m*r) # theta
deux points
```

```
    return [dr,dtheta,dv,dw]
```

```
t = np.linspace(0, 4000000, 1000000) # intervalle de
temps de l'étude
```

```
x0= 6800000 #altitude initiale
```

```
v0= 0
```

```
theta = pi/2
```

```
omega = 7635/6800000 # vitesse angulaire initiale
en rad/s
```

```
# tableau des conditions initiales
```

```
Cl=np.array([x0,theta,v0,omega])
```

```
Sols=odeint(fonction,Cl,t)
```

```
r= Sols[:, 0]
```

```
u = Sols[:, 1]
```

```
# courbe de l'altitude par rapport au temps
```

```
plt.plot(t,-6371000+r, linewidth=0.25, color='m')
```

```
plt.grid()
```

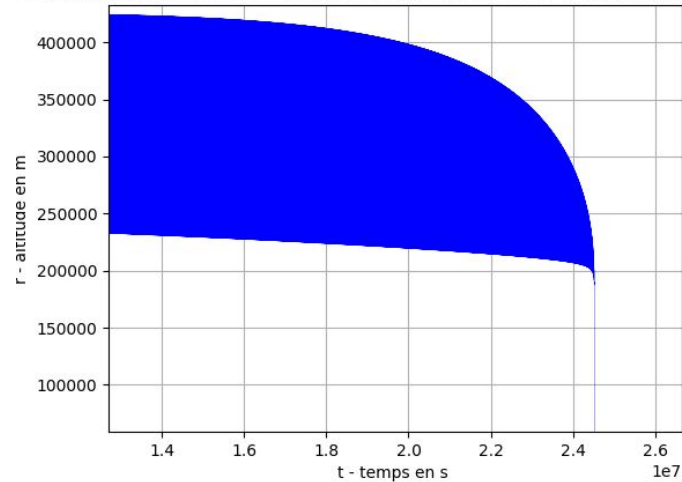
```
plt.xlabel('t - temps en s')
```

```
plt.ylabel('r - altitude en m')
```

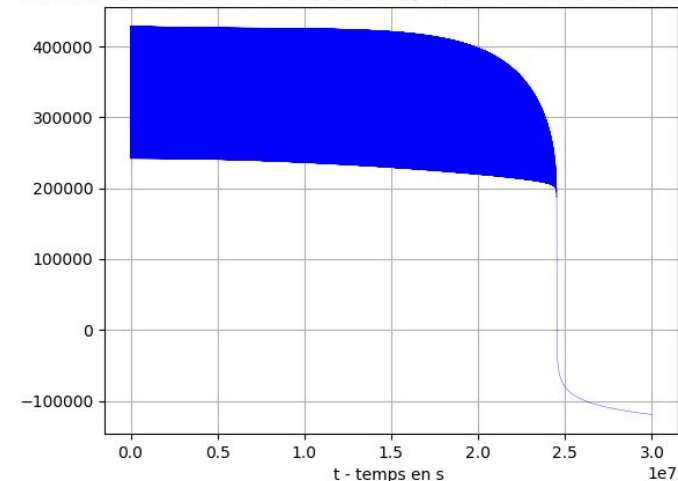
```
plt.title(" L'altitude du satellite en fonction du temps
(avec force de radiation)")
```

```
plt.show()
```

L'altitude du satellite en fonction du temps (avec force de radiation  $F=0.01I$ )



L'altitude du satellite en fonction du temps (avec force de radiation  $F=0.01I$ )



## Part of the program: Altitude versus time without solar shading

Appendix slides

SSAT=50 # la surface change

def fonction(X,t):

    r = X[0]

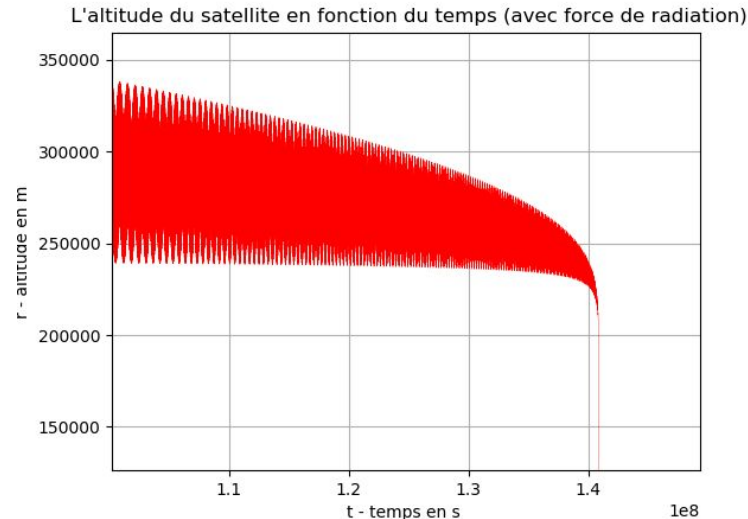
    theta = X[1]

    v = X[2]

    omega = X[3]

    dr = v # r point

    dtheta = omega #theta point



$$dv = -(G \cdot M) / (r^{**2}) + r \cdot (dtheta^{**2}) + (\exp((6371000-r)/hs) \cdot SSAT \cdot \sqrt{dr^{**2} + (r \cdot dtheta)^{**2}}) \cdot (Cp \cdot r \cdot dtheta - Ct \cdot dr) / (2 \cdot m) - F \cdot \sin(theta) / m$$
 # r deux points

$$dw = -2 \cdot dr \cdot dtheta / r - (\exp((6371000-r)/hs) \cdot SSAT \cdot \sqrt{dr^{**2} + (r \cdot dtheta)^{**2}}) \cdot (Ct \cdot r \cdot dtheta - Cp \cdot dr) / (2 \cdot m \cdot r) - (F \cdot \cos(theta)) / (m \cdot r)$$
 # theta deux points

return [dr,dtheta,dv,dw]



## Part of the program: end-of-life satellite trajectory

Appendix slides

```
t = np.linspace(-3000000, 3000000, 1000000) # intervalle de temps de l'étude
```

```
x0= 6800000 #altitude initiale
```

```
v0= 0
```

```
theta = pi/2
```

```
omega = 7635/6800000
```

```
# tableau des conditions initiales
```

```
Cl=np.array([x0,theta,v0,omega])
```

```
Sols=odeint(fonction,Cl,t)
```

```
# traçage de la terre
```

```
theta = np.linspace(0, 2*np.pi, 1000)
```

```
A = 6371000*np.cos(theta)
```

```
B = 6371000*np.sin(theta)
```

```
plt.plot(A, B)
```

```
x = Sols[:, 0]
```

```
u = Sols[:, 1]
```

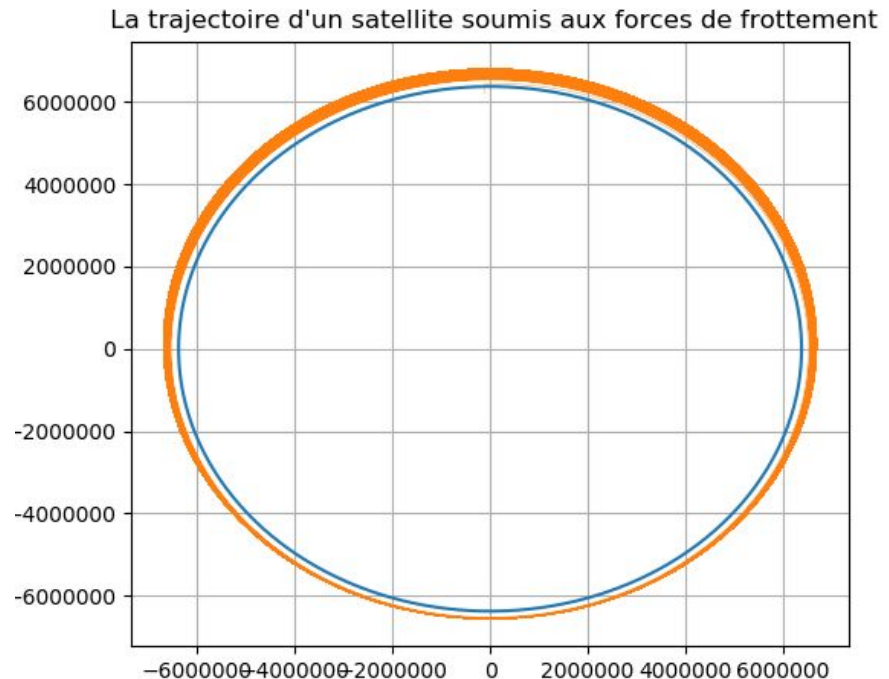
```
# courbe de la trajectoire
```

```
plt.plot(x*cos(u),x*sin(u), linewidth=0.25 )# projection sur x et y pour obtenir la  
courbe finale
```

```
plt.grid()
```

```
plt.title( " La trajectoire d'un satellite soumis aux forces de frottement" )
```

```
plt.show()
```



## Program section: Altitude versus time for Nanosail-D2

Appendix slides

**#introduction des constantes et les caractéristiques de ce satellite**

m=4

SSAT=8

Cp=0.349

Ct= 1.34 **# coefficients qui dépendent normalement de la forme du satellite**

Mair= 28.976\* 10\*\*(-3)

g=9.81

R=8.31

T=273.15

hs =(R\*T)/(Mair\*g)

**def** fonction(X,t):

    r = X[0]

    theta = X[1]

    v = X[2]

    omega = X[3]

    dr = v

    dtheta = omega

**if** theta % 2\*pi > pi:   **# la force de radiation ne s'exerce pas sur la voile dans la zone d'ombre**

    F=0

**else:**

        F=0.00009 **# nouvelle force calculée pour S= 10 m2**

        dv = -(G\*M)/(r\*\*2) + r\*(dtheta\*\*2) + (  
exp((6371000-r)/hs) \*SSAT\* sqrt(dr\*\*2  
+(r\*dtheta)\*\*2)\*(Cp\* r\* dtheta - Ct  
\*dr))/(2\*m)-F\*sin(theta)/m **# r deux points**

        dw = - 2\*dr\*dtheta/r - ( exp((6371000-r)/hs)  
\*SSAT \*sqrt(dr\*\*2 +(r\*dtheta)\*\*2)\*(Ct\* r\* dtheta - Cp  
\*dr) )/(2\*m\*r)-(F\*cos(theta))/(m\*r) **# theta deux points**

    return [dr,dtheta,dv,dw]



```
t = np.linspace(0, 25000000, 1000000)
```

```
x0= 7002500 #Moyenne du périgée 615km et l'apogée 648 km
```

```
v0= 0
```

```
theta = pi/2
```

```
omega = 2* pi /(97.34*60) # période 97.34min
```

```
# tableau des conditions initiales
```

```
Cl=np.array([x0,theta,v0,omega])
```

```
Sols=odeint(fonction,Cl,t)
```

```
r= Sols[:, 0]
```

```
u = Sols[:, 1]
```

```
plt.plot(t,-6371000+r, linewidth=0.25, color='m')
```

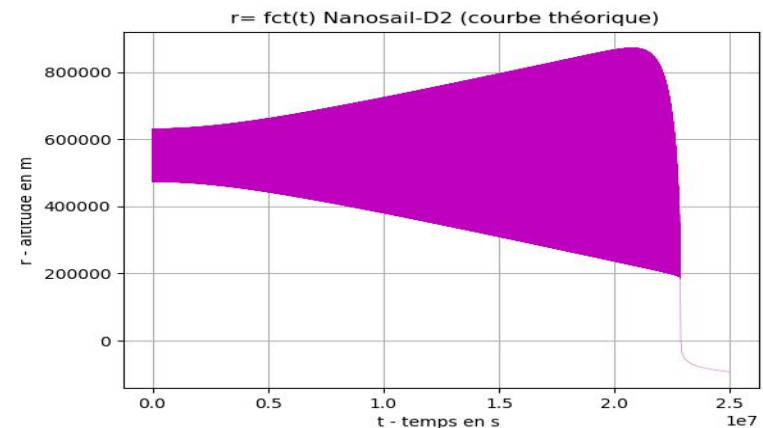
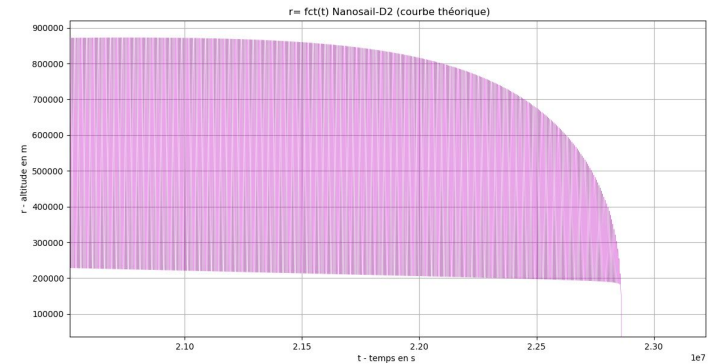
```
plt.grid()
```

```
plt.xlabel('t - temps en s')
```

```
plt.ylabel('r - altitude en m')
```

```
plt.title(" r= fct(t) Nanosail-D2 (courbe théorique)")
```

```
plt.show()
```



## Orbit duration as a function of sail area

```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
```

```
X=[0,600, 1200,3600,6000]
```

```
Y=[1620 , 566.8 , 284, 95 , 56.92]
```

# utilisation du programme “altitude en fonction du temps” pour plusieurs valeurs de F

```
plt.plot(X,Y, color='k', marker='o')
```

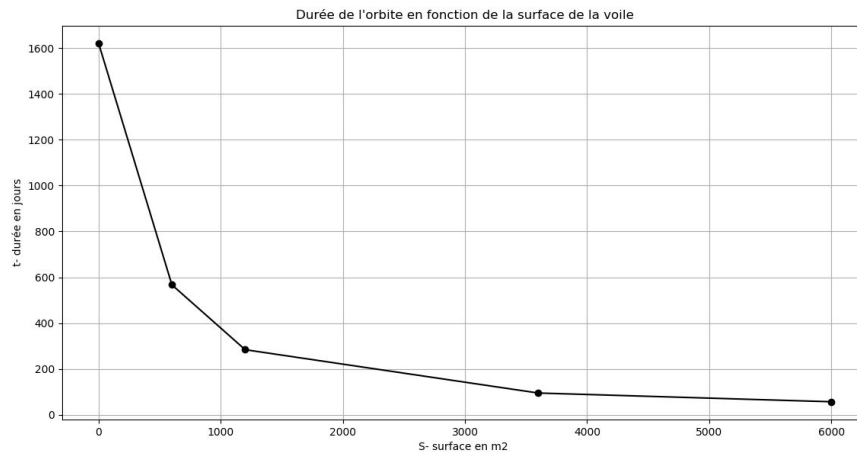
```
plt.title("Durée de l'orbite en fonction de la surface de la voile")
```

```
plt.xlabel('S- surface en m2 ')
```

```
plt.ylabel('t- durée en jours')
```

```
plt.grid()
```

```
plt.show()
```



## Approximation with Regressi:

