INF TC1 - Lecture 3: Trees

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In [2]:
```
import sys
import os
from graphviz import Digraph
from IPython.display import display
from utils import draw_tree_dict
```

Outline

- Definitions
- Data structures
- Weighted trees

Trees

> *Tree is a hierarchical data structure with nodes connected by edges*

- A non-linear data structures (multiple ways to traverse it)
- Nodes are connected by only one path (a series of edges) so trees have no cycle
- Edges are also called links, they can be traversed in both ways (no orientation)

Example of trees:

- Binary trees, binary search trees, N-ary trees, recursive call trees, etc.
- HOB (Horizontally Ordered Binary), AVL (Adelson-Velskii and Landis, self-balancing trees), ...
- B-trees, forests, lattices, etc.

Definitions on trees

(similar to the ones for the binary trees)

`Nodes` - a tree is composed of nodes that contain a `value` and `children`.

`Edges` - are the connections between nodes; nodes may contain a value.

`Root` - the topmost node in a tree; there can only be one root.

`Parent and child` - each node has a single parent and up to two children.

`Leaf` - no node below that node.

`Depth` - the number of edges on the path from the root to that node.

`Height` - maximum depth in a tree.

Definitions on trees (cont.)

`N-ary Tree` - a tree in which each node can have up to N children. Binary trees is the case where $N = 2$.

`Weight` - a quantity is associated to the edges.

`Degree` - the number of child nodes it has. Binary tree is the case where degree is 2.

`Subtree` - a portion of a tree that is itself a tree.

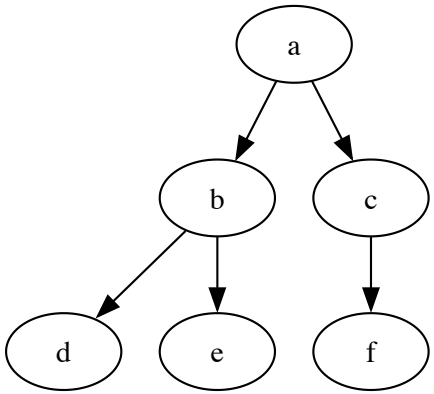
`Forest` - a collection of trees not connected to each other.

Data structures (dicts + lists)

A simple way is the adjacency list using a dictionnary `dict` type.

In [3]:
```
tree_dict = {
    "a": ["b", "c"],
    "b": ["d", "e"],
    "c": ["f"],
    "d": [],
    "e": [],
    "f": []
}
```

In [4]:
```
draw_tree_dict(tree_dict)
```



Data structures (dicts + named lists)

A variation is to use a named variable for the list.

In [6]:
```
tree_dict_name = {
    "a": {"neighbors": ["b", "c"]},
    "b": {"neighbors": ["d", "e"]},
    "c": {"neighbors": ["f"]},
    "d": {"neighbors": []},
    "e": {"neighbors": []},
    "f": {"neighbors": []}
}
```

In [7]:
```
tree_dict_name["a"]["neighbors"]
```

Out[7]:
```
['b', 'c']
```

Data structures (sets)

The children are unique and not ordered.

In [8]:
```
tree_set = {
    "a": set(["b", "c"]),
    "b": set(["d", "e"]),
    "c": set(["f"]),
    "d": set(),
    "e": set(),
    "f": set()
}
```

Data structures (lists of lists)

- Each node is an entry in the list
- Children are sub-lists

In [9]:
```
tree_list = [
    ['a', ['b', 'c']],
    ['b', ['d', 'e']],
    ['c', ['f', 'g']],
    ['d', []],
    ['e', []],
    ['f', []],
    ['g', []]
]
```

Data structures (tuples)

- Each node is the first tuple
- Children are additionnal tuply entries
- Warning: tuples are immutable (cannot be changed)

In [10]:
```
tree_tuple = ("a", [
    ("b", []),
    ("c", [
        ("d", [
            ("e", [])
        ])
    ])
])
```

In [11]:
```
tree_tuple[0] # cannot be changed
```

Out[11]:
```
'a'
```

Data structure (class object)

How to create the tree? How to retrieve all nodes? Both iterative and recursive ways.

```
class Node:
    def __init__(self, value, children = []):
        self.value = value
        self.children = children
```

Data structure (class object)

How to create the tree? How to retrieve all nodes? Both iterative and recursive ways.

```
class Node:
    def __init__(self, value, children = []):
        self.value = value
        self.children = children
```

In [12]:

```
class Node:
    def __init__(self, value, children = []):
        self.value = value
        self.children = children

    def get_all_nodes(self):
        nodes = [self.value]
        for child in self.children:
            nodes += child.get_all_nodes()
        return nodes

    def get_all_nodes_iterative(self):
        nodes = []
        stack = [self]
        while stack:
            current_node = stack.pop()
            nodes.append(current_node.value)
            stack += current_node.children
        return nodes
```

In [13]:
```
root = Node("a", [
    Node("b", [
        Node("d"),
        Node("e"),
    ]),
    Node("c", [
        Node("f"),
    ]),
])

# or using "root.children"
```

In [14]:
```
root.get_all_nodes()
```

Out[14]:
```
['a', 'b', 'd', 'e', 'c', 'f']
```

In [15]:
```
root.get_all_nodes_iterative()
```

Out[15]:
```
['a', 'c', 'f', 'b', 'e', 'd']
```

Weighted trees

> *Trees with a quantity associated to the links or the nodes*

- Useful to quantifie both nodes and links
- Storing those values require additionnal data structures

Data structures for weighted trees (dicts for edges)

- We need to add an extra value to encode values in edges

In [16]:
```
tree_w_dict = {'a': [{'b': 0}, {'c': 0}],
        'b': [{'d': 0}, {'e': 0}],
        'c': [{'f': 0}],
        'd': [],
        'e': []
        }
```

In [17]:
```
tree_w_tuple = {
    'a': [('b', 0), ('c', 0)],
    'b': [('d', 0), ('e', 0)],
    'c': [('f', 0)],
    'd': [],
    'e': []
}
```

Weigthted trees as classes

In [18]:
```
class Node_weight:
    def __init__(self, data, weight=0):
        self.data = data
        self.children = []
        self.weight = weight


tree = Node_weight(1)
child1 = Node_weight(2, weight=5)
child2 = Node_weight(3, weight=7)
tree.children = [child1, child2]
```

Exercise: Calculate the total weight of a tree

Tip: go through all the nodes and get the edges, then sum their weights.

Exercise: Calculate the total weight of a tree

Tip: go through all the nodes and get the edges, then sum their weights.

In [19]:
```python
def get_tree_edges(root):
    edges = []
    stack = [(root, None)]

    while stack:
        node, parent_data = stack.pop()

        for child in node.children:
            stack.append((child, node.data))
            edges.append((node.data, child.data, child.weight))

    return edges
```

In [20]:
```python
tree_w_oo = Node_weight(1)
child1 = Node_weight(2, weight=5)
child2 = Node_weight(3, weight=7)
tree_w_oo.children = [child1, child2]
get_tree_edges(tree_w_oo)
```

Out[20]:
```
[(1, 2, 5), (1, 3, 7)]
```

In [22]:
```python
sum(tpl[2] for tpl in get_tree_edges(tree_w_oo)) # 12
```

Out[22]: 12

Exercise: Calculate the total weight of a tree

A recursive version:

In [23]:
```
def calculate_total_weight(node):
    total_weight = node.weight
    for child in node.children:
        total_weight += calculate_total_weight(child)
    return total_weight
```

In [24]:
```
calculate_total_weight(tree_w_oo)
```

Out[24]:
```
12
```

An Edge class for edges

- To consider edges as objects
- Can be used as a complement of the nodes (or without the nodes)

In [25]:
```
class Edge:
    def __init__(self, source, target):
        self.source = source
        self.target = target

class Node:
    def __init__(self, label):
        self.label = label
        self.children = []

class Tree:
    def __init__(self, root_label):
        self.root = Node(root_label)
        self.edges = []
```

Check trees properties

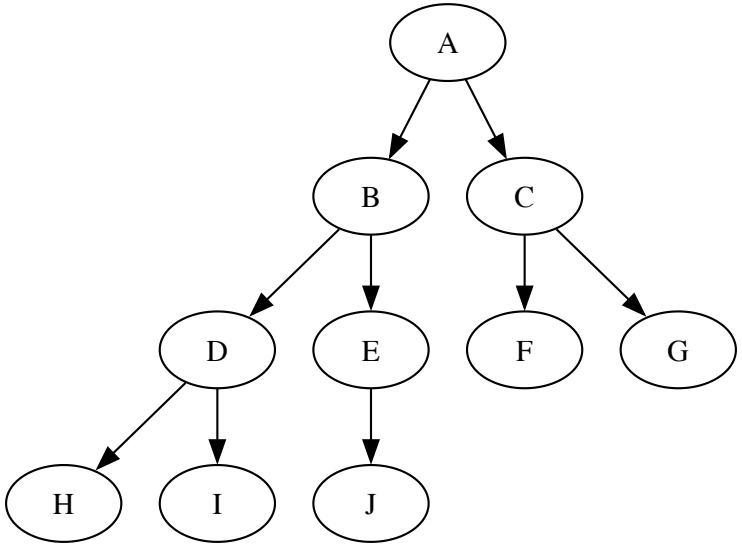
- Hierarchical structure
- No cycle
- All nodes connected

We will mostly use one of the two traversal methods (BFS and DFS) to achieve this.

Also we will using the dictionnary-based data structure:

In [26]:
```
tree = {
    "A": ["B", "C"],
    "B": ["D", "E"],
    "C": ["F", "G"],
    "D": ["H", "I"],
    "E": ["J"],
    "F": [],
    "G": [],
    "H": [],
    "I": [],
    "J": []
}
```

In [27]:
```
draw_tree_dict(tree)
```


A
B
C
D
E
F
G
H
I
J

Generalized BFS (Breadth-First Search)

In [28]:
```python
def bfs(tree, start_node):
    queue = [start_node]
    result = []

    while queue:
        node = queue.pop(0)
        result.append(node)
        children = tree.get(node, [])

        for child in children:
            if child is not None:
                queue.append(child)

    return result
```

In [29]:
```python
print(bfs(tree, "A"))
```

```
['A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J']
```

Generalized DFS (Depth-First Search)

In [30]:
```python
def dfs(tree, start_node):
    stack = [start_node]
    result = []

    while stack:
        node = stack.pop()
        result.append(node)
        children = tree.get(node, [])

        for child in children:
            if child is not None:
                stack.append(child)

    return result
```

In [31]:
```python
print(dfs(tree, "A"))
```

```
['A', 'C', 'G', 'F', 'B', 'E', 'J', 'D', 'I', 'H']
```

Tree property: are all nodes connected?

Without having a first node and re-using the dfs

In [34]:
```python
def dfs_check_connected(tree, start_node):
    if not tree:
        return True  # an empty tree is considered connected.

    visited = set()
    stack = []

    stack.append(start_node)

    while stack:
        node = stack.pop()
        if node not in visited:
            visited.add(node)
            stack.extend(tree.get(node, []))

    return len(visited) == len(tree)


dfs_check_connected(tree, "A")
```

Out[34]:
```
True
```

Tree property: does the tree have a cycle?

In [38]:

```
def has_cycle(graph, start_node):
   visited = set()
   parent = {}

   def dfs(node):
       visited.add(node)

       for neighbor in graph.get(node, []):
           if neighbor in parent and parent[node] == neighbor:
               continue

           if neighbor in visited:
               return True

           parent[neighbor] = node

           if dfs(neighbor):
               return True

       return False

   return dfs(start_node)
has_cycle(tree, "A")
```

Out[38]:

```
False
```

What if we add an extra node ”K”?

tree["F"] = ["A"]

In [40]:
```
tree["F"] = ["A"]
has_cycle(tree, "A")
```

Out[40]:
```
True
```

Tree property: Check if the tree is an n-ary tree

In [41]:

```
def is_binary_tree(tree, node, n = 2, visited=None):
    if visited is None:
        visited = set()

    if node in visited:
        return True

    visited.add(node)
    children = tree.get(node, [])

    if len(children) > n:
        return False

    for child in children:
        if not is_binary_tree(tree, child, n, visited):
            return False

    return True

is_binary_tree(tree, "A", 2)
```

Out[41]: True

Get all the edges of a tree

In [42]:
```
def generate_edges(graph):
    edges = []
    for node, neighbors in graph.items():
        for neighbor in neighbors:
            edges.append((node, neighbor))
    return edges
```

In [43]:
```
generate_edges(tree)
```

Out[43]:
```
[('A', 'B'),
 ('A', 'C'),
 ('B', 'D'),
 ('B', 'E'),
 ('C', 'F'),
 ('C', 'G'),
 ('D', 'H'),
 ('D', 'I'),
 ('E', 'J'),
 ('F', 'A')]
```

In [44]:
```python
def generate_edges_dfs(graph, start_node):
    edges = []
    stack = [start_node]
    visited = []

    while stack:
        node = stack.pop()
        visited.append(node)
        for neighbor in graph[node]:
            if neighbor not in visited:
                edges.append((node, neighbor))
                stack.append(neighbor)

    return edges
```

In [45]:
```python
generate_edges_dfs(tree, "A")
```

Out[45]:
```
[('A', 'B'),
 ('A', 'C'),
 ('C', 'F'),
 ('C', 'G'),
 ('B', 'D'),
 ('B', 'E'),
 ('E', 'J'),
 ('D', 'H'),
 ('D', 'I')]
```