Programming strategies

> *A programming strategy are algorithms aimed at solving a specific problem in a precise manner.*

Examples of Strategies:

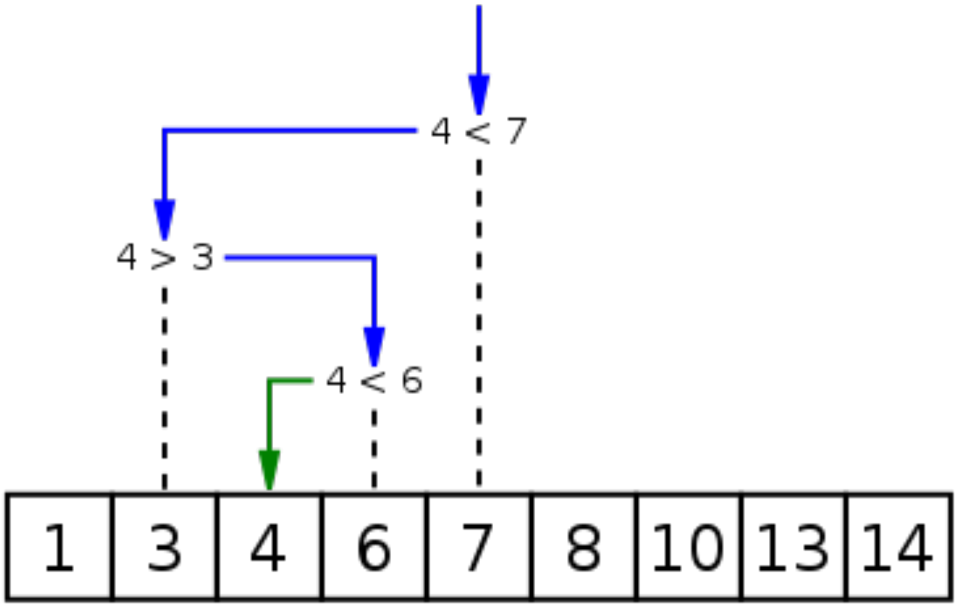
- **Divide and Conquer:** Divide a problem into simpler sub-problems, solve the sub-problems, and then combine the solutions to solve the original problem.
- **Dynamic Programming:** Solve a problem by breaking it down into sub-problems, calculating and memorizing the results of sub-problems to avoid unnecessary recomputation.
- **Greedy Algorithm:** Make a series of choices that seem locally optimal at each step to find a solution, with the hope that the result will be globally optimal as well.

Examples of divide and conquer algorithms:

- Binary search
- Quick sort and merge sort
- Map Reduce
- Others: Fast multiplication (Karatsuba)

Binary search

Given a sorted list, find or insert a specific value while keeping the order.

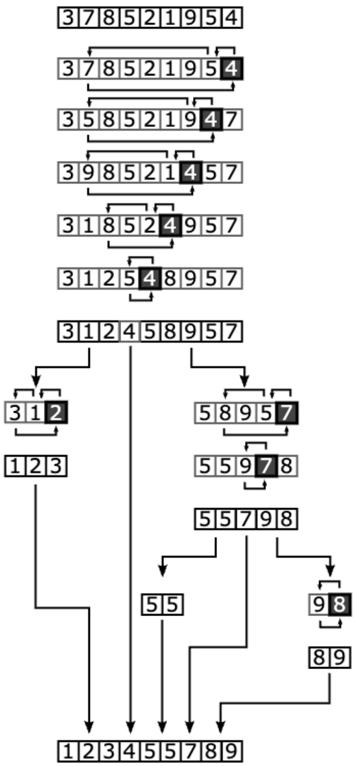


See the notebook.

Quick sort

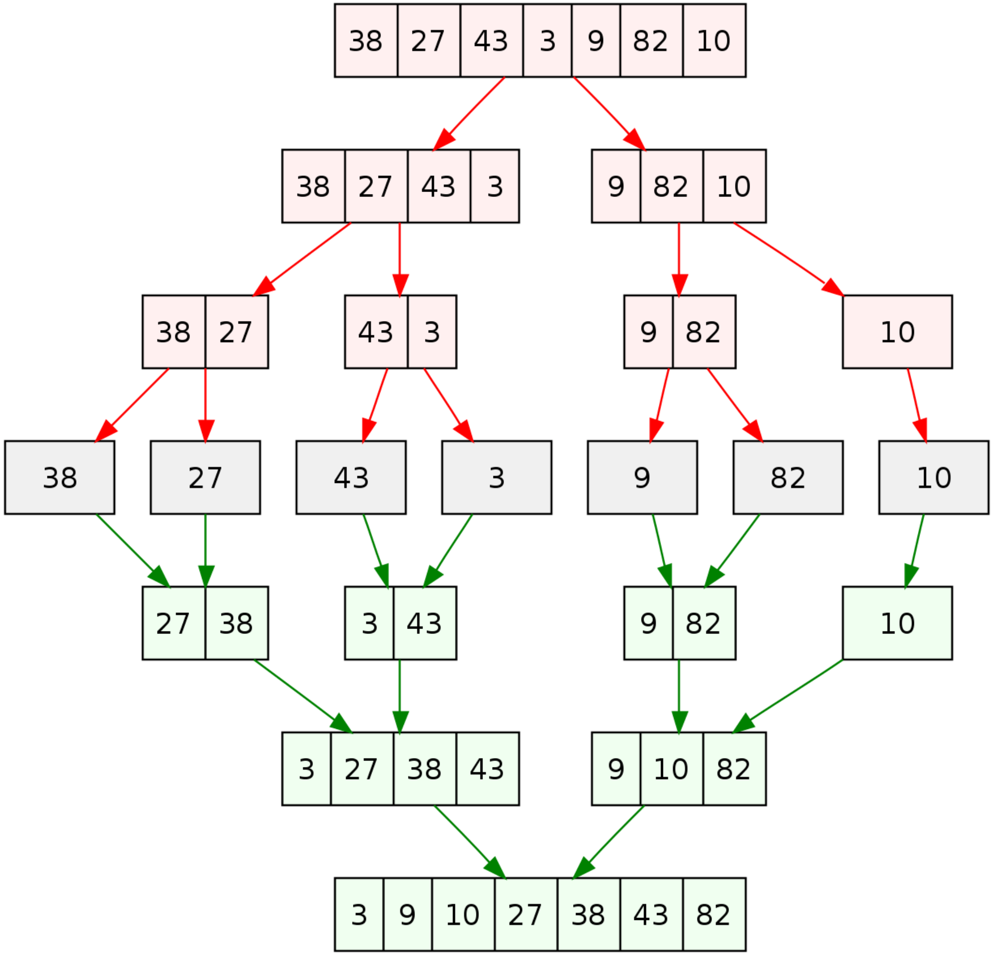
Recursive sorting algorithm which works in two steps:

1. select a pivot element
2. partitioning the array into smaller sub-arrays, then sorting those sub-arrays.



Merge sort

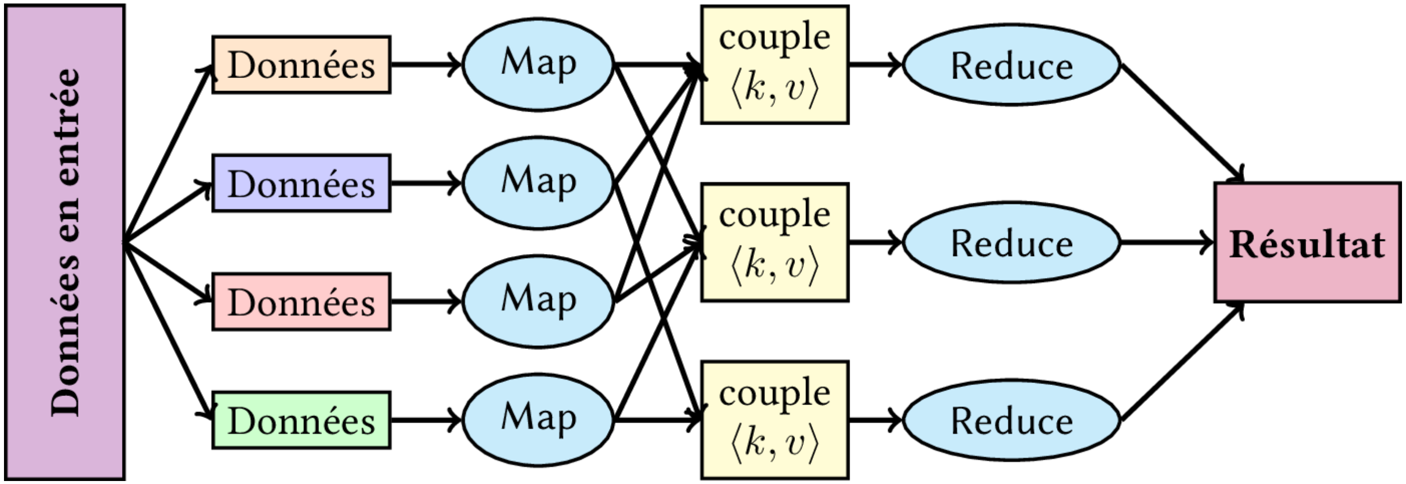
Divide an array recursively into two halves (based on a *pivot* value), sorting each half, and then merging the sorted halves back together. This process continues until the entire array is sorted.
Complexity: $O(nlog(n))$.



Map reduce

Divide a large dataset into smaller chunks and processes them independantly. Two main steps:

- the Map stage, where data is filtered and transformed into key-value pairs
- the Reduce stage, where data is aggregated and the final result is produced.



Map reduce (without map reduce..)

Calculate the sum of squares values from a list of numerical values.

In [49]:
```
data = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

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In [49]:
```
data = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

In [60]:
```
result = {}
for num in data:
    square = num * num
    result[square] = num

final_result = list(result.items())

print(final_result)
print(sum([x[0] for x in final_result]))
```

```
[(1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6), (49, 7), (6
4, 8), (81, 9), (100, 10)]
385
```

Map reduce (Python)

1. Divide the problem in sub-problems
2. Apply the mapping function
3. Reduce the results

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In [2]:

```python
data = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

def mapper(numbers):
    result = []
    for num in numbers: # calculate the squares
        result.append((num, num * num))
    return result

def reducer(pairs):
    result = {}
    for key, value in pairs: # sums the squares
        if key in result:
            result[key] += value
        else:
            result[key] = value
    return result.items()
```

Map reduce (Python)

1. Divide the problem in sub-problems
2. Apply the mapping function
3. Reduce the results

In [21]:

```
chunk_size = 2
chunks = [data[i:i+chunk_size] for i in range(0, len(data), chunk_size)

mapped_data = [mapper(chunk) for chunk in chunks]

grouped_data = {}# map
for chunk in mapped_data:
    for key, value in chunk:
        if key in grouped_data:
            grouped_data[key].append(value)
        else:
            grouped_data[key] = [value]

reduced_data = [reducer(list(grouped_data.items()))] # reduce
print(sum([x[1][0] for x in list(reduced_data[0])]))
```

```
385
```

Discussion on Divide and Conquer

- Similarities with recursion by dividing a problem in a sub-problem
- But with a combination step (which may hold most of the code difficulty)
- Can be implemented in a non-recursive way
- $nlog(n)$ complexity when split the problem and solves each split

Example: Change-making problem

$Q_{opt}(S, M) = min\ \sum_{i=1}^{n} x_i.$

S: all the available coins

M: amount

Greedy solution:

1. Sort the coins in descending order
2. Initialize a variable to count coins used
3. Substrack the number of coins used (if limited)
4. Continue this process until amount becomes zero.

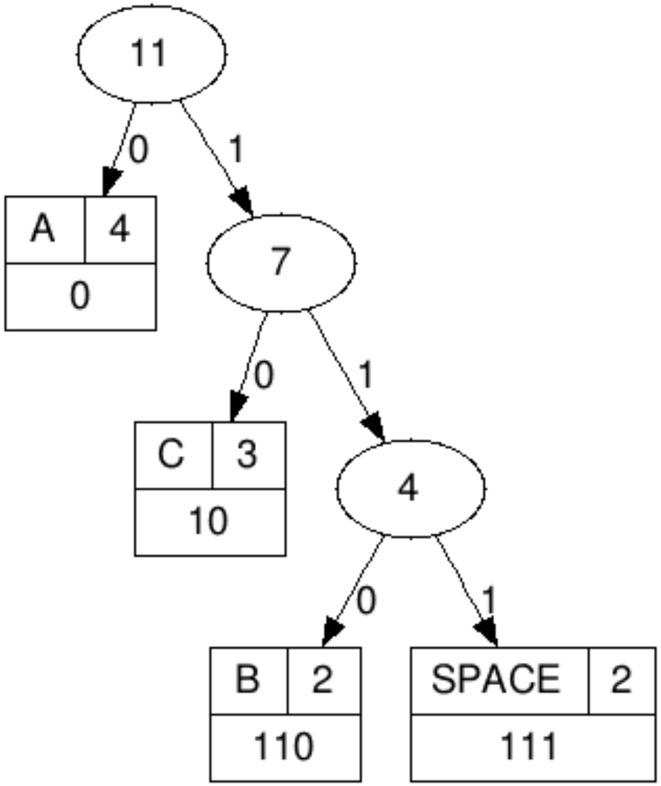
Example: Huffman code

A prefix code for lossless information compression. The key idea is to build a character frequency tree, and then traverse the tree to obtain the code (`0` for left branches, `1` for right branches).

Steps to Follow:

1. Create a queue with letter frequencies.
2. Take the two least frequent characters from the list.
3. Create a node in a tree that contains the sum of frequencies and the two characters as children. Add this node to the queue and remove the children.
4. Repeat steps 2 and 3 until no more elements remain; the last element is the root of the tree.

Example: Huffman code



Discussion on Greedy algorithms

- Often considered as an *heuristic*
- Easy to understand, implement and communicate
- They often lead to non-optimal solution

Examples of dynamic programming algorithms

- Fibonacci Sequence
- Rod Cutting
- Sequence Alignment, Longest Subsequence Finding
- Shortest Path Finding

Fibonnacci (reminder)

To calculate the n-th number in the Fibonacci sequence, which is determined as follows:

latex Copy code $fib(n) = fib(n-1) + fib(n-2)$, $n \in \mathbb{N}$ Where the sequence starts with 1, 1, and then continues as 2, 3, 5, 8, 13, 21, and so on, to find the 9th number ($n = 9$).

Let's calculate the 9th Fibonacci number step by step:

$fib(1) = 1$

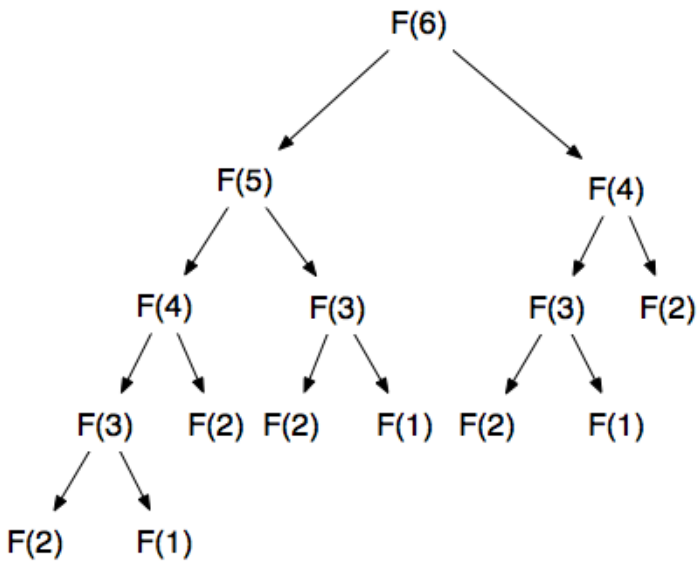
$fib(2) = 1$

$fib(3) = fib(2) + fib(1) = 1 + 1 = 2$

Fibonnacci (naive)

In [4]:
```
def fib(n):
    if n < 2:
        return n
    else:
        return fib(n - 1) + fib(n - 2)
```

Call tree (for $n = 6$):



Requires to calculate the same F-value multiple times.

Fibonnacci (dynamic programming)

Optimized using a `lookup` *table, which is a data structure to memoize values that have already been computed.*

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In [97]:
```
def fib(n, lookup):
    if n == 0 or n == 1:
        lookup[n] = n

    if lookup[n] is None:
        lookup[n] = fib(n - 1, lookup) + fib(n - 2, lookup)

    return lookup[n]

def main():
    n = 6

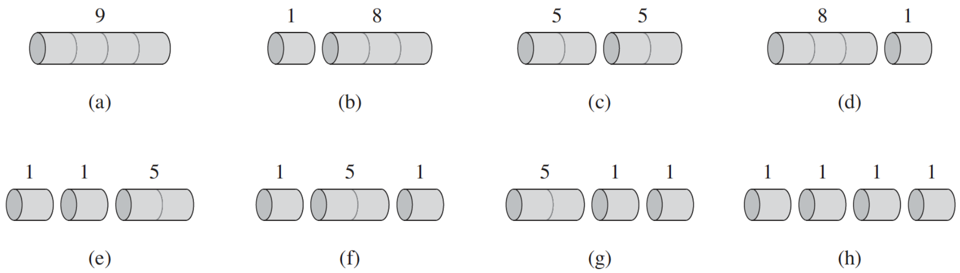
    lookup = [None] * (n + 1)
    result = fib(n, lookup)
    print(f"{n}-th Fibonacci number is {result}")

if __name__=="__main__":
    main()
```

```
6-th Fibonacci number is 8
```

Rod cutting

Given a list of cuts and prices, identify the optimal cuts. Given the example below, what is the best cutting strategy for a rod of size 4 ?



size (i)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
price (pi)	1	5	8	9	10	17	17	20

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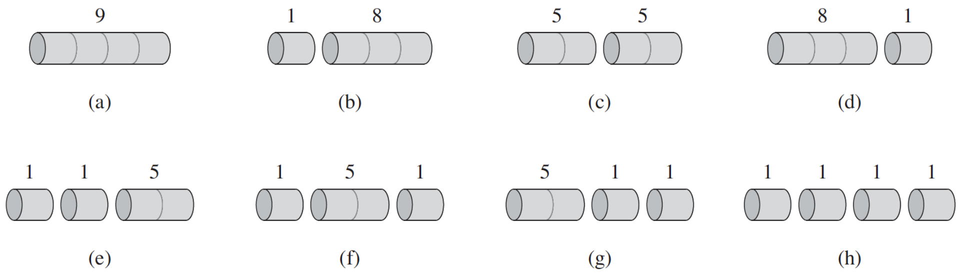

size (i)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
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Solution: For a rod of size 4 optimal solution is 2 cuts of size 2 so .

Rod cutting: check a solution

Given the previous table of size and price, check the cost of a given solution by defining a function `check_rod_cutting(prices, n)`.

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In [27]:

```
def check_rod_cutting(prices, n):
    table = [0] * (n + 1)

    for i in range(1, n + 1):
        max_price = float('-inf')
        for j in range(1, i + 1):
            max_price = max(max_price, prices[j] + table[i - j])
        table[i] = max_price

    return table[n]
```

In [30]:

```
prices = [0, 1, 5, 8, 9, 10, 17, 17, 20]
n = 2

max_total_price = check_rod_cutting(prices, n)
print(f"The maximum total price for a rod of length {n} is {max_total_p
```

```
The maximum total price for a rod of length 2 is 5
```

Rod cutting (brute force)

Let's solve the rod cutting problem using a brute force (naive) approach.

1. define a value function
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In [33]:
```
def cut_brute_force(n, t):
    if n == 0:
        return 0
    max_valeur = float('-inf')
    for i in range(1, n + 1):
        valeur_courante = t[i] + coupe_brute_force(n - i, t)
        max_valeur = max(max_valeur, valeur_courante)
    return max_valeur
```

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        max_valeur = max(max_valeur, valeur_courante)
    return max_valeur
```

In [34]:

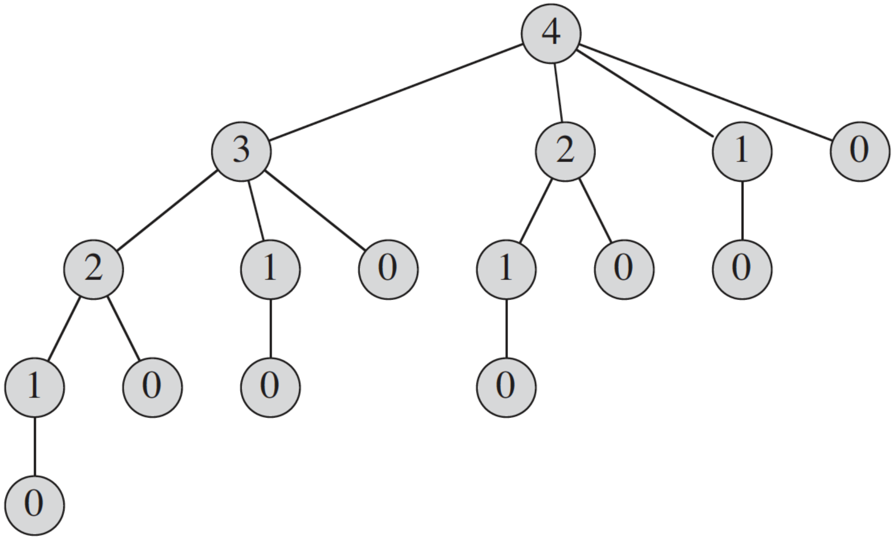
```
lengths = [0, 1, 2, 3, 4, 5, 6, 7, 8]
values = [0, 1, 5, 8, 9, 10, 17, 17, 20]
rod_length = 2
max_value = coupe_brute_force(rod_length, values)
print(f"The maximum value for a rod of length {rod_length} is {max_valu
```

```
The maximum value for a rod of length 2 is 5.
```

Rod cutting (dynamic programming)

General case:

- Cutting a rod of length i optimally.
- Cutting a rod of length $(n - i)$ optimally.



General case: $V_n = max_{1 \leq i \leq n}(p_i + V_{n-i})$

size (i)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
price (pi)	1	5	8	9	10	17	17	20

$V_3 = \max_{1 \leq i \leq 3}(p_i + V_{3-i})$

Let's calculate `V_3` step by step for each possible value of `i`:

1. If `i = 1`, we cut the rod into two pieces: one of length 1 and one of length 2.
    - $V_1 = p_1 = 2$
    - $V_{3-1} = V_2$
2. If `i = 2`, we cut the rod into two pieces: one of length 2 and one of length 1.
    - $V_2 = p_2 = 5$
    - $V_{3-2} = V_1$
3. If `i = 3`, we cut the rod into one piece of length 3.
    - $V_3 = p_3 = 9$
    - $V_{3-3} = V_0$ (Assuming that $V_0 = 0$ as a base case.)

Now, we can calculate the values for V_2 and V_1 recursively using the same formula:

For V_2 :

$$V_2 = \max(p_1 + V_1, p_2 + V_0) = \max(2 + V_1, 5 + 0) = \max(2 + 2, 5 + 0) = \max(4, 5) = 5$$

For V_1 :

$$V_1 = \max(p_1 + V_0) = \max(2 + 0) = 2$$

So, V_2 is 5 and V_1 is 2.

Now, we can calculate V_3 using the values of V_2 and V_1 :

$$V_3 = \max(p_1 + V_2, p_2 + V_1, p_3 + V_0) = \max(1 + 5, 5 + 2, 9 + 0) = \max(6, 7, 9) = 0$$

Rod cutting (dynamic programming)

Rod cutting (dynamic programming)

In [36]:

```python
INT_MIN = 0

def cutRod(price, n):

    # init cache tables
    val = [0 for x in range(n+1)]
    val[0] = 0

    for i in range(1, n+1):
        max_val = INT_MIN
        for j in range(i):
            max_val = max(max_val, price[j] + val[i-j-1])
        val[i] = max_val

    return val[n]

if __name__=="__main__":
    arr = [1, 5, 8, 9, 10, 17, 17, 20]
    size = len(arr)
    print("Max size cut " + str(cutRod(arr, size)), len(arr) )
```

```
Max size cut 22 8
```

Lessons on dynamic programming

- It is necessary to study each problem on a case-by-case basis.
- Storing a large number of partial results, which requires significant memory usage.
- Suitable for only certain problems (min, max, counting the number of solutions).